Closing *Tues*: 4.4-5 (graphing)
Closing *Thurs*: 4.7 (applied max)
Final: Sat, Dec. 9, 1:30-4:20pm, Kane 130
Assigned seats, for your seat go to:
catalyst.uw.edu/gradebook/aloveles/102715

4.7 Applied Max/Min

Entry Task: Get out the handout from last time.

How to approach applied max/min:

1. Visualize/Label.
2. Constraints: What are we given?
3. **Objective: What are we optimizing?**
4. Get a *one variable* function for the value we are optimizing.
5. Engage your calculus muscles.
6. Justify your answer.

Straight from homework example:
Find two numbers whose difference is 188 and whose product is minimum.

x = smaller number
y = larger number
GIVEN: $y - x = 188 \Rightarrow y = 188 + x$
WANT: MINIMIZE THE PRODUCT!
PRODUCT = xy

$$f(x) = x(188 + x) = 188x + x^2$$

$$f'(x) = 188 + 2x \stackrel{?}{=} 0$$

$$2x = -188$$

$$x = -94 \Rightarrow y = 94$$

JUSTIFY

1st deriv test

f ↘ ↗

f' − -94 +

ONE CRITICAL PT
LOCAL MIN
⇒ GLOBAL MIN

2nd deriv test

$f'' = 2$

ONE CRITICAL PT
CONCAVE UP
⇒ LOCAL MIN
⇒ GLOBAL MIN

ENDPOINTS!

$\lim_{x \to \infty} f(x) = \infty$

$\lim_{x \to -\infty} f(x) = \infty$

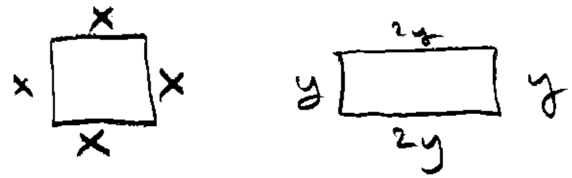
ONE CRITICAL PT
$x = -94$
MUST BE MINIMUM

Straight from Old Final example:

A farmer has 136 meters of fencing. She wants to make two rectangular enclosures. One will be a square. The other will have its long side twice as long as its short side (Allow the possibility that all of the fencing could go to only one of the enclosures.)

What dimensions will make the combined total area as small as possible?

What dimensions will make the combined total area as big as possible?



GIVEN: $4x + 6y = 136 \Rightarrow$ $6y = 136 - 4x$, $y = \frac{136}{6} - \frac{2}{3}x$

$\frac{68}{3}$

WANT: OPTIMIZE COMBINED AREA.

$$\text{AREA} = x^2 + 2y^2$$

$$f(x) = x^2 + 2\left(\frac{68}{3} - \frac{2}{3}x\right)^2$$

$$f'(x) = 2x + 4\left(\frac{68}{3} - \frac{2}{3}x\right)\cdot\left(-\frac{2}{3}\right)$$

$$= 2x - \frac{8}{9}(68 - 2x)$$

$$= 2x - \frac{544}{9} + \frac{16}{9}x \overset{?}{=} 0$$

$$\Rightarrow 18x - 544 + 16x = 0$$

$$34x = 544$$

$$x = \frac{544}{34} = 16$$

$$y = \frac{68}{3} - \frac{2}{3}(16) = 12$$

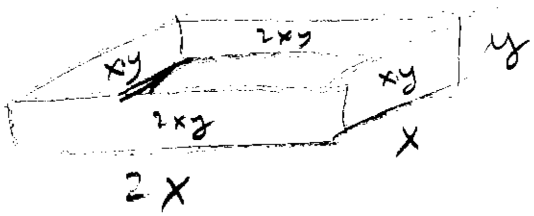
ENDPOINTS: $x = 0$ to $4x = 136 \Rightarrow x = 34$

$x = 0 \Rightarrow \text{AREA} = f(0) = 2\cdot\left(\frac{68}{3}\right)^2 = \frac{9248}{9} \approx 1027.\overline{5}\ m^2$

$x = 16 \Rightarrow \text{AREA} = f(16) = 16^2 + 2(12)^2 = \boxed{544\ m^2 \leftarrow \text{MIN}}$

$x = 34 \Rightarrow \text{AREA} = f(34) = 34^2 + 2(0)^2 = \boxed{1156\ m^2 \leftarrow \text{MAX}}$

Straight from homework example:
A box with a rectangular base and open top must have a volume of 10 cm³. The length of one side of the rectangle is twice the width. The material for the base costs $5.00 per square meter and the material for the sides costs $3.00 per square meter. Find dimensions and the corresponding cost for the cheapest container.



GIVEN: VOLUME $= (2x)(x)(y) = 2x^2y = 10 \Rightarrow y = 5/x^2$

WANT: MINIMIZE $\underline{\text{COST}}$

$$\text{COST} = 5\begin{pmatrix}\text{AREA OF}\\ \text{BASE}\end{pmatrix} + 3\begin{pmatrix}\text{AREA}\\ \text{OF}\\ \text{SIDES}\end{pmatrix}$$

$$= 5(2x \cdot x) + 3(2xy + 4xy)$$

$$= 10x^2 + 18xy$$

$$f(x) = 10x^2 + 18x\left(\frac{5}{x^2}\right)$$

$$= 10x^2 + \frac{90}{x}$$

$$f'(x) = 20x - \frac{90}{x^2} \stackrel{?}{=} 0$$

$$20x^3 - 90 = 0$$

$$x^3 = \frac{90}{20}$$

$$x = (9/2)^{1/3} \approx 1.65096$$

$$y = \frac{5}{(9/2)^{2/3}} = \frac{5}{9/2}(9/2)^{1/3}$$

$$= \frac{10}{9}\left(\frac{9}{2}\right)^{1/3}$$

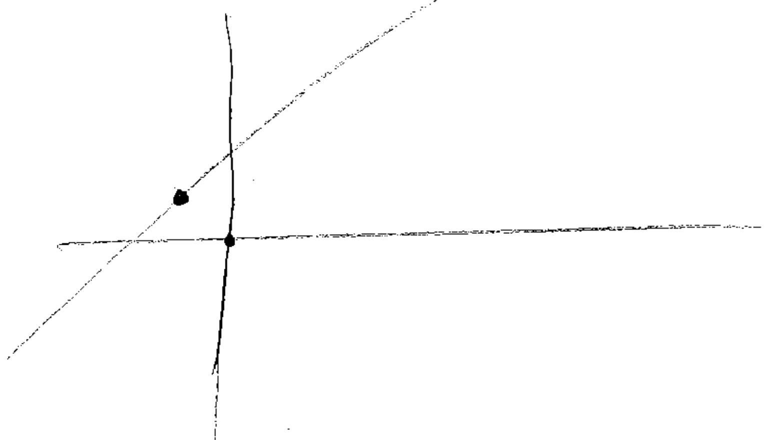
$$\approx 1.8344$$

f	↘		↗		$f'' = 20 + \frac{180}{x^3}$
f'	$-$	1.65	$+$		$+$

ONE CRITICAL PT & LOCAL MIN.

Straight from homework example:

Find the point on the line y = 4x+3 that is closest to the origin.



GIVEN: (x,y) ON $y=4x+3$

WANT: MINIMIZE DISTANCE

FROM (x,y) to $(0,0)$

$$\text{DIST} = \sqrt{(x-0)^2+(y-0)^2}$$

$$= \sqrt{x^2+y^2}$$

$$f(x) = \sqrt{x^2+(4x+3)^2}$$

$$f'(x) = \frac{1}{\cancel{2}\sqrt{\text{mess}}}\left(\cancel{2}x+\cancel{2}(4x+3)4\right) \stackrel{?}{=} 0$$

$$f'(x) = \frac{1}{\sqrt{\text{mess}}}(17x+12)$$

$$x+16x+12 \stackrel{?}{=} 0$$

$$17x = -12$$

$$x = -\frac{12}{17}$$

$$y = 4\left(-\frac{12}{17}\right)+3$$

$$= -\frac{48}{17}+3 = 3/17$$

$$(x,y) = \left(-\frac{12}{17}, \frac{3}{17}\right)$$

f: ↘ ↗

f': $-$ $\quad -\frac{12}{17} \quad$ $+$

ONE CRITICAL PT & LOCAL MIN

$\Rightarrow$ GLOBAL MIN.